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recursive contracts

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Abstract

A number of dynamic models in economics are formulated with forward-looking elements in the constraints – for example, models of risk-sharing with participation constraints and models of optimal policy. Here, standard dynamic programming does not apply. Recent contributions show how to reformulate these models by either rewriting the forward-looking constraints (promised utility approach) or by using a Lagrangean formulation (recursive Lagrangean). Both make it possible to obtain a recursive formulation that allows for easier computation and analytical results. A number of applications can be found to optimal fiscal or monetary policy, risk sharing or investment with various financial constraints, and employment decisions.

Keywords

Bellman equation; commitment; contract theory; debt constraints; dynamic programming; incentive constraints; international capital flows; Lagrange multipliers; optimal fiscal policy; optimal monetary policy; optimal taxation; participation constraints; principal and agent; private information; Ramsey equilibria; recursive contracts; risk sharing; saddle point functional equations; time consistency; unemployment insurance

Article

In contract theory it is standard to introduce a participation constraint (PC) insuring that the contract offered to the agent delivers a utility higher than the best outside option. In a dynamic set-up agents may abandon the contract at any point in time, even after the contract has been in place for a while. For example, workers can leave a labour contract at almost no cost, or a borrower can stop repaying the loan if he or she declares bankruptcy. The possibility that the agent does not continue with the plan of the contract is usually called 'default'. Hence, in a dynamic context, it is natural to require that the PC is satisfied in all periods, in order to avoid default.

It turns out that, if a PC in all periods and realizations is introduced in the design of the optimal contract, standard dynamic programming does not apply, the Bellman equation does not hold, and the solution is not guaranteed to be a time-invariant function of the usual state variables. This complicates enormously the solution of these models.

To discuss this in a simple risk-sharing model, consider two agents i=1,2 with utility function $E_0\sum_{t=0}^{\infty}\beta^t u(c_t^i)$, where $\beta\in(0,1)$ is the discount factor and u the instantaneous utility. Each agent receives a stochastic endowment w_t^i and the realization of endowments is known both to the agents and the principal. The principal has full commitment, and will stick to his announced plan. Endowments provide the only supply of consumption good so that the following feasibility condition holds

$$c_t^1 + c_t^2 = w_t^1 + w_t^2 \tag{1}$$

A Pareto-optimal risk-sharing contract (implemented by a competitive equilibrium under complete markets) would set $\frac{u'(c_t^1)}{u'(c_t^2)}$ constant for all periods, so that agents would share all idiosyncratic risks. This allocation would be chosen as the optimal contract if agents would commit to never leave the risk-sharing arrangement. We refer to this allocation as the first best. The optimum satisfies the usual recursive structure in dynamic models, namely, that c_t=$F(w_t)$ where F is a time-invariant function and $w_t = (w_t^1, w_t^2)$.

Assume now agents cannot commit to staying in the contract for ever. An agent can leave the contract and consume for ever his individual endowment, so that a contract can only be implement if it satisfies

$$E_t\sum_{j=0}^{\infty}\beta^j u(c_{t+j}^i) \geq V_i^a(w_t)$$

at all periods and realizations, where $V_i^a(w_t) \equiv E_t\sum_{j=0}^{\infty}\beta^j u(w_{t+j}^i)$ is the utility of consuming in autarchy for ever after t.

It is clear that the above PC is likely to be violated by the first best allocation. In periods when w_t^i is high, the right side of the PC is high, but the agent has to surrender a large part of his endowment in the first best and the left side of the PC is too low. Therefore, PCs are often binding and they make the first best unfeasible.

A Pareto-optimal risk-sharing contract with PCs can now be found by maximizing the weighted utility of the two agents $E_0\sum_{t=0}^{\infty}\beta^t[\lambda u(c_t^1 c) + (1-\lambda)u(c_t^2)]$ subject to the above PC for all periods and realizations and for both agents. The parameter λ indexes all such Pareto-optimal allocations. The result is an optimal contract under full commitment by the principal and partial commitment by the agents.

The Bellman equation does not give the solution to this problem. A key feature of standard dynamic programming is that the set of feasible actions

must depend only on variables that were determined last period and the current shock. But it is possible to evaluate if a certain consumption level $\bar{c}_t^i$ satisfies the PC at time t only if future plans for consumption are known.
Intuitively, a promise of higher consumption in the future makes a lower consumption today compatible with the PC. But in order to implement this plan the principal has to ‘remember’ all the promises for higher consumption that were made in the past. Therefore, the optimal solution is unlikely to be a function of only today's endowment, the principal also needs to recall if, say, ten periods ago, the PC of one of the agents was binding.
As argued by Kydland and Prescott (1977), the same problem arises in models of optimal policy. The future restricts today's actions through the first order conditions of optimality of the agents, this causes the Bellman equation to fail and, in their language, the solution was time inconsistent. We find the same difficulty in contracting models of private information with incentive constraints, where some relevant piece of information is hidden from the principal, and more generally, in game theoretical models where an agent optimizes subject to the plans for the future of another agent.
If the Bellman equation fails, the solution could depend on all past shocks, and solving for the variables as a function of all past shocks would be very difficult. Too many variables would appear as arguments of the decision function. To overcome this difficulty the ‘recursive contracts’ literature provides several alternatives. The general idea is to recover a recursive formulation by adding a co-state variable.
One approach builds on the paper of Abreu, Pierce and Stachetti (1990; hereafter APS). To show how this can be applied in the above risk-sharing model with PCs, consider the case where w_t is i.i.d. and has two possible realizations $\bar{w}$ and $\bar{\bar{w}}$ with probabilities π and $(1-\pi)$. Denote the utility of agent i for the whole future at t if $w_t = \bar{w}$ by $\bar{V}_t^i \equiv E_t(\sum_{j=0}^{\infty}\beta^j u(c_{t+j}^i)|w_t = \bar{w})$, and let $\bar{\bar{V}}_t^i$ be the analogue for realization $\bar{\bar{w}}$. The above PC can be reformulated as

$$V_t^i = u(c_t^i) + \beta\left(\pi\bar{V}_{t+1}^i + (1-\pi)\bar{\bar{V}}_{t+1}^i\right)$$

(2)

$$\bar{V}_{t+1}^i \geq V_i^a(\bar{w}),\quad \bar{\bar{V}}_{t+1}^i \geq V_i^a(\bar{\bar{w}})\ \text{ for all }\ t > 0,$$

where V_t^i is the actual realized utility. The first equation insures that V_t^i is the expected discounted utility, the second guarantees that the PC holds. We can view the planner's choice at t as choosing the promised utilities $\bar{V}_{t+1}^i$, $\bar{\bar{V}}_{t+1}^i$ and consumption c_t^i, while V_t^i is given by past choices. It is clear that, in the APS approach, today's choice variables $x_t = (\bar{V}_{t+1}^i, \bar{\bar{V}}_{t+1}^i, c_t^i)$ are restricted by yesterday's promised utilities only, and the Bellman equation delivers the optimal contract after the realized V_t^i is included in the list of state variables. The promised utility V_t^i plays the same role as capital in a standard growth model, and (2) plays the role of the transition equation. Therefore, the optimal solution for the choices can be described recursively by a time-invariant function $x_t = F(w_t, V_t)$ for all $t>0$.
A crucial caveat is that (2) is not sufficient to insure that the PCs are satisfied. The principal could choose arbitrarily high consumption and have ever higher Vs to satisfy (2), in a sort of Ponzi scheme for utility. The promised utilities have to be further restricted to belong to a feasible set. Let us call $\bar{\boldsymbol{S}} \subset R$ the feasible set of utilities such that, for each element $v \in \bar{\boldsymbol{S}}$, there is a sequence of consumptions $\left\{c_{t+j}^i\right\}$ that satisfy (1) and the PCs such that $v = E_t(\sum_{j=0}^{\infty}\beta^j u(c_{t+j}^i)|w_t = \bar{w})$. Results in APS insure that this set is convex. Since in this case $\bar{\boldsymbol{S}} \subset R$, this set is an interval and there exist bounds $\bar{V}_L^i$ and $\bar{V}_U^i$ such that adding the constraints

$$\bar{V}_L^i \leq \bar{V}_{t+1}^i \leq \bar{V}_U$$

(and similarly for $\bar{\bar{V}}_t^i$) to (2) is enough to insure feasibility. These bounds can be easily introduced in the Bellman equation and this guarantees that the chosen consumption sequences satisfies the PC. The only complication is that upper bound $\bar{V}_U$ needs to be computed separately, as it is not a datum of the problem ($\bar{V}_L^i$ is trivially equal to $V_i^a(\bar{w})$).
Another difference with standard dynamic programming is that the initial utility V_0^1 is an outcome of the solution and it is not fixed beforehand. This feature shows how time inconsistency arises in this model, since the choice for V in period zero is not given, but in future periods it is given from the past.
Promised utilities as co-states have been used extensively in models with incentive or participation constraints. Among others, Phelan and Townsend (1991) studied a model of risk-sharing with incentive constraints, Kocherlakota (1996) analysed the risk-sharing model with the PC described above, Hopenhayn and Nicolini (1997) a model of unemployment insurance and Alvarez and Jermann (2000) a decentralized version of the above risk-sharing model with debt constraints. In models of Ramsey equilibria it has been used by Golosov, Kocherlakota and Tsyvinski (2003) to study optimal taxation under private information and Chang (1998) in a model of optimal monetary policy.
The main problem with this approach is the computation of the set of feasible utilities $\bar{\boldsymbol{S}}$. In the specific model described above this is not too costly, because it involves finding only two numbers, namely, the upper bounds $\bar{V}_U$, $\bar{\bar{V}}_U$. But the difficulties multiply when more than one co-state variable is needed. For example, if a third agent is included in the above risk-sharing model, the co-state variables would be (V_t^1, V_t^2). Results in APS guarantee that the set of feasible utilities $\bar{\boldsymbol{S}} \subset R^2$ is convex, but now it is a generic set, not an interval. Computing a set is much harder than computing two numbers. Some papers overcome these difficulties; for example, Abraham and Pavoni (2005), who show how to find such a set in a model of saving under private information, or the paper of Judd, Yeltekin and Conklin (2003). But the difficulties increase very fast with the

dimensionality of the promised utilities.
Furthermore, in some models, the set of feasible promised utilities changes every period. If a ‘traditional’ state variable (say, capital stock) appears in the problem, the set of feasible utilities is different depending on the level of capital, so that the feasible set is now given by a correspondence $\bar{\boldsymbol{S}}(k)$. The researcher now needs to solve for a mapping from capital stock to sets. Phelan and Stacchetti (2001) compute in this way the optimal fiscal policy in a model with capital.
An alternative to APS is the Lagrangean approach described in Marcet and Marimon (1998). The Lagrangean for the optimal risk-sharing problem with PC is

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda u(c_t^1) + (1-\lambda) u(c_t^2) + \sum_{i=1,2} \gamma_t^i \left(E_t \sum_{j=0}^{\infty} \beta^j u(c_{t+j}^i) - V_i^a(w_t) \right) \right]$$

where $\gamma_t^i \geq 0$ is the Lagrange multiplier of the PC. This can be rewritten as

$$L = E_0 \sum_{t=0}^{\infty} \beta^t [(\lambda + \mu_t^1) u(c_t^1) + (1 - \lambda + \mu_t^2) u(c_t^2)] \text{ s.t. } \mu_t^i = \mu_{t-1}^i + \gamma_t^i,\ \gamma_t^i \geq 0 \quad \mu_{-1}^i = 0$$

In this formulation, only current and past variables enter in the objective and in the constraints of this Lagrangean, and a proper initial condition for μ is given. In this approach, μ_t plays the role of the co-state variable instead of the promised utility in the APS approach. A saddle point functional equation (analogous but not equal to the Bellman equation) is satisfied, insuring that the optimal solution satisfies $(c_t,\gamma_t)=G(\mu_{t-1},w_t)$ with $\mu_{-1}^i = 0$ for a time invariant function G.
The equilibrium satisfies $\frac{u'(c_t^1)}{u'(c_t^2)} = \frac{1-\lambda+\mu_t^2}{\lambda+\mu_t^1}$. If the PC for agent i is binding, the corresponding γ_t^i is strictly positive, the weight μ_t^i goes up and so does c_t^i. The increase in μ is permanent (at least until another PC is binding). In this way the principal avoids default by spreading the reward over time in order to enhance smoothing of consumption.
Note that the initial value of μ is given and equal to zero, while in future periods μ_{t-1} needs to be set according to past Lagrange multipliers.
Therefore, the initial value of the co-state does not need to be found separately as in APS. It is clear that, if the principal could re-optimize ignoring past commitments at sometime t, he or she would ignore the past co-state and reset μ=0. This is how time inconsistency is reflected in this formulation.
In the Lagrangean approach there is no need to find the set of feasible utilities. The only constraint on the co-states is the non-negativity constraint on γs. Application to models with capital accumulation and several co-states is much easier; for example, Marcet and Marimon (1992) solve a risk-sharing growth model with PC as described above and capital accumulation, Aiyagari et al. (2002) in a Ramsey equilibrium for fiscal policy under incomplete markets, where debt is a state variable, Attanasio and Ríos-Rull (2000) risk-sharing in small villages, Scott (2007) a model of optimal taxes with capital, Kehoe and Perri (2002) international capital flows with capital accumulation under PC, King, Kahn and Wolman (2003) optimal monetary policy, Cooley, Marimon and Quadrini (2004) a model of investment under private information, Abraham and Carceles-Poveda (2006) discuss how to decentralize a model with participation constraints, and Ferrero and Marcet (2004) and Scholl (2004) a model of temporary exclusion in the case of default. The drawback of the Lagrangean approach is that, at this writing, the theory for the non-convex case and for the private information case is still incomplete.

See Also

- agency problems
- Bellman equation
- dynamic programming
- income taxation and optimal policies
- optimal fiscal and monetary policy (with commitment)
- optimal taxation
- risk sharing
- time consistency of monetary and fiscal policy

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